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On the penetration of a turbulent layer into stratified fluid

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An experiment is described in which a constant stress is applied to the surface of an initially quiescent tank of fluid with a uniform density gradient. The development of the turbulent layer by entrainment of the underlying fluid is described and it is found that the entrainment coefficient E, the ratio of the entrainment velocity u_e to the friction velocity u_* is given in terms of the depth D of the mixed layer and the density jump $\delta\rho$ across the entrainment interface by the relation

$$E = \frac{u_e}{u_*} = 2{\cdot}5\,\frac{\rho_0 u_*^2}{g\,\delta\rho\,D}.$$

The rate of increase of potential energy of the stratified fluid was found to be proportional to the rate of dissipation of kinetic energy per unit area in the turbulent layer. The form of these results is consistent with those found by Turner with an agitation tank, but the parameters used here allow direct application to entrainment in the ocean.

1. Introduction

In the ocean, under calm conditions, the upper twenty or thirty metres usually exhibit a continuous, moderately stable density distribution. When a wind begins to blow over the surface, turbulence in the water is generated both by the mean shear and by the sporadic breaking of waves. As time goes by, the turbulent layer becomes deeper as a result of the entrainment or erosion by the turbulence of underlying denser water. Because of the relatively rapid mixing, the density distribution is approximately uniform in the upper layer, and the entrainment takes place across an 'interface' between the turbulent and non-turbulent fluids over which the density difference increases as the process proceeds. On a smaller scale and at greater depths, it is frequently found that the density field in the ocean consists of a sharp succession of well-mixed layers of constant density separated by sharp, gravitationally stable interfaces. (See, for example, the recent summaries by Cooper 1967 and Stommel & Federov 1967.) The vertical transport of such properties as heat and salt in these regions of the ocean is evidently governed by their transfer across the strongly stable interfaces.

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There has been a number of very valuable experimental studies on turbulent entrainment across a density jump. Rouse & Dodu (1955) studied the rate of entrainment in an interface between two superimposed layers of fluid of slightly different densities, when turbulence is generated in the upper layer by mechanical agitation. Cromwell (1960), Turner & Kraus (1967) and Turner (1968), using essentially similar but successively refined apparatus, have succeeded in clarifying many aspects of this process but there remains the inherent difficulty with experiments of this kind that the structure and appropriate scales of the turbulence generated by the mechanical agitation are far from obvious and difficult to measure. Though interesting and consistent results can be obtained from agitation experiments, it is sometimes rather difficult to interpret them in an oceanographical context, where the turbulence is generated in other ways. The aim of the present study is to explore the possibility of generating a turbulent layer not by direct stirring but by the application of a known horizontal stress at the upper ('free') surface of a stratified fluid and to make preliminary measurements of the rate at which the layer penetrates into the underlying fluid. The surface stress, applied at time $t = 0$ and then held constant, and the depth of the layer provide two simple parameters with which to characterize the large scale properties of the turbulence.

In the ocean, of course, the wind stress acts over a very large area of the surface. It seems hardly possible to simulate such conditions accurately even in a long laboratory tank because of the end effects and the necessity for a return flow of the stratified fluid to compensate for the mass transport induced by the surface stress. We sought to overcome this difficulty by the use of a moderately large, non-rotating annular tank. Though avoiding the problem of end effects, this arrangement has the possible disadvantage that unwanted secondary motions might be developed in the turbulent region as the fluid, accelerated by the surface stress, moves around the annulus. This matter will be considered later in the light of the experimental findings; suffice it at this stage to point out that in a stably stratified region, secondary motions are naturally inhibited, and the centrifugal effect of a mean streaming motion around the annulus would be expected merely to produce a small radial tilt of the surfaces of constant density.

2. Apparatus

The apparatus consisted, in essence, of an annular tank that could be filled with fluid, at the surface of which a constant stress could be applied in the circumferential direction. The tank was constructed of galvanized iron, the outer and inner diameters being 152·4 cm and 106·7 cm respectively, so that the width of the channel was 22·8 cm. These dimensions were maintained to a tolerance of $\pm$ 0·2 cm at most. The total depth of the tank was 28·0 cm. It was mounted on a plywood base, supported by a concrete and steel platform, the whole being adjusted carefully so that the base was horizontal. The tank was fitted with two windows. One, on the outer side, through which observations were made, was formed from transparent plastic $\frac{1}{4}$ in. thick, curved and set carefully to provide continuity with the cylindrical inner surface of this side of the tank. At the same section, a second window of flat glass was installed in the floor to allow illumination of the flow for visual observations and photography. At first, an ordinary soda glass window was used, but some difficulty was experienced with convective motions in this region driven by the heating from the lamp, particularly when the stratification was weak. With the replacement of the ordinary glass by a piece of 'thermopane' glass and with the consistent use of a cooling fan, this difficulty disappeared; all measurements were taken after these modifications had been made. It was desired to illuminate only the central region of the channel, so that a thin vertical sheet of light was produced by a simple optical system consisting of a high intensity photographic lamp, aluminium plates to provide a narrow slit and a cylindrical lens to collimate the beam.

The surface stress was applied by the wheel arrangement shown in figure 1 (plate 1) from which hung a plastic screen that could be raised and lowered from the water surface. The wheel was mounted to spin freely about a vertical shaft, and attached to the lower side by an extension of the wheel hub was a large driving sprocket. In its rest position (with the screen in contact with the water surface) the hub extension rested on a thrust roller bearing, just below the sprocket. This is shown in figure 2. At the upper end of the wheel hub was a collar beneath which the forked end of a lever could be inserted to lift this whole assembly an inch or two in the vertical, so that the screen was raised above the water level. The wheel was driven by a chain from a small sprocket attached to the gear box of a small electric motor. It was important in the experiment to be able to measure the total torque exerted on the water surface, so the motor and gear box were mounted on a turntable, coaxial with the wheel, that could also rotate freely on roller bearings but independently of the wheel assembly. To measure the torque, the motor platform was restrained by a light spring between the turntable itself and the fixed base; when the angular velocity of the wheel was constant, the torque applied to the water surface was balanced by that supplied through the spring to the turntable, this being the only point of restraint of the motor-platform, wheel system.

In order to minimize the torque required for small changes in the angular velocity of the wheel, it was constructed as lightly as possible so that its moment of inertia was as small as could reasonably be achieved. From the rim of the wheel the screen was suspended by a series of wire hangers attached to a frame of light aluminium tubing (figure 1, plate 1). The plastic screen (flyscreen woven with a rectangular mesh, the threads being about 0·02 cm in diameter with 0·14 cm spacing) was cut from two large pieces, the joint being glued carefully, and the cut screen stretched tautly over the lower side of the frame. The clearances between the edges of the frame and the sides of the tank were about 0·4 cm. Great care was taken in adjusting the arrangement of wheel, hangers and screen so that the height of the screen did not vary by more than $\pm 0{\cdot}2$ cm as it rotated; the light construction made it difficult to do much better than this despite many attempts. During operation, the screen was immersed to an average depth of 0·7 cm, the cross struts being clear of the water.

The shearing stress applied to the water surface was measured by the angular displacement θ of the turntable restrained by the spring. The equation of motion of the rotating system can be represented as

$$I\ddot{\phi} + k\dot{\phi} = T - T_w, \tag{1}$$

where ϕ is the angular displacement of the wheel, I the moment of inertia of the wheel and screen, k a frictional constant of the supporting bearings, T the torque applied, measured by the extension of the spring and T_w the torque given to the water. The friction of the bearings was evaluated by running the screen steadily

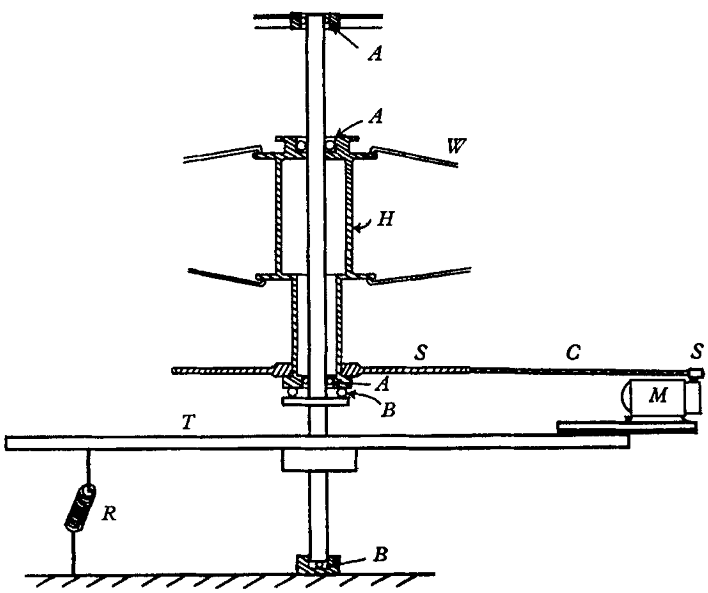

FIGURE 2. A sketch of the mechanism. The turntable T is mounted on a central shaft, supported by a thrust bearing B and a roller bearing at the top A; it is restrained by the light spring R. The motor M is mounted on the turntable. It drives the wheel assembly W by means of the sprockets S, S and chain C. The wheel assembly, constructed on the hub H, is free to rotate about the central shaft, being supported by the thrust bearing B and ball bearings A, A.

without water, when the torque T is simply $k\dot{\phi}$. At the speeds of operation used, this was found to be too small to be measured. Efforts were made to ensure that the effect of the first term on the left-hand side was also negligible. The moment of inertia of the rotating system was measured to be approximately 5×10^7 g cm^2. In a typical experiment, the angular acceleration was largest during the 4 or 5 seconds after starting the motion; after 15 seconds or so it did not exceed 5×10^{-3} rad/sec^2 and decreased as the experiment proceeded (figure 3). During this time, the magnitude of the first term in (1) was $2{\cdot}5 \times 10^5$ c.g.s. units at most. The torques measured were an order of magnitude larger, so that inertial effects were neglected. Consequently,

$$T = T_w = 2\pi \int_a^b r^2 \tau \, dr,$$

where a and b are the inner and outer radii of the screen and τ the surface stress. If τ_0 is the average stress applied to the surface,

$$T_w = \tfrac{1}{2}(a+b)\,A\,\tau_0\left\{1+O\left(\frac{b-a}{2a}\right)^2\right\},$$

where A is the screen area. With the dimensions used, the second term was of the order 5 per cent and neglected, the surface stress being taken as

$$\tau \simeq \tau_0 = \frac{2T_w}{(a+b)A}. \qquad (2)$$

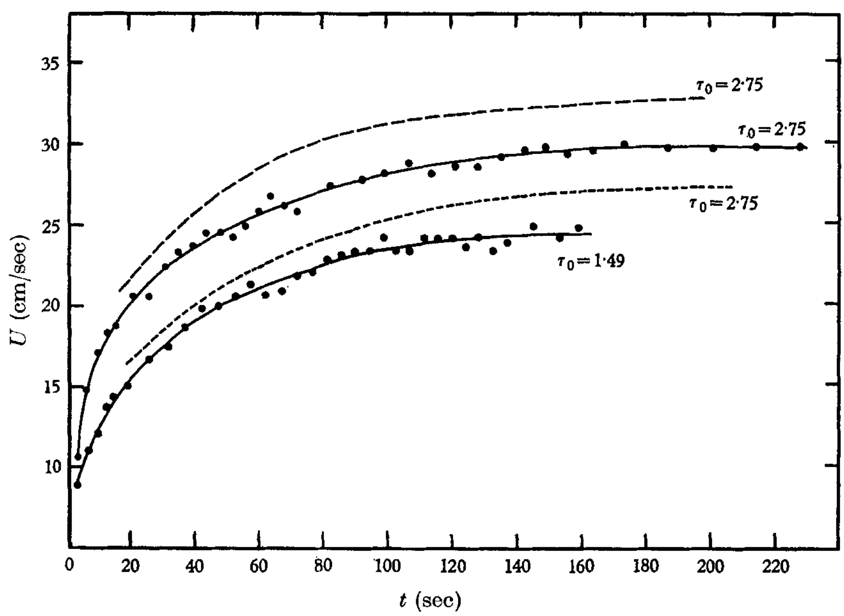

FIGURE 3. Examples of variations in screen speed U with time. – – –, $d\rho/dz = 0{\cdot}00192$ g cm^{-4}; ———, $d\rho/dz = 0{\cdot}00384$ g cm^{-4}; — — —, $d\rho/dz = 0{\cdot}00769$ g cm^{-4}.

A static calibration was made by measuring the displacement θ of the turntable under the application to the circumference of the wheel of a tangential force by means of an attached string passing over a pulley and carrying weights. The applied torques were expressed in terms of the equivalent surface stress by use of (2); it was found that the relation between τ and θ was very nearly linear over the range used (figure 4).

In all the experiments, the tank was filled to a depth of 23·0 cm. The linear density gradient was produced by allowing to diffuse a series of layers with successive increases in density (Mowbray 1967). For all runs described in this paper, twenty layers were used, each of depth 1·15 cm. The appropriate volume of fresh water was let slowly into the bottom of the tank by the use of a ring of copper tubing with many small holes along its bottom, that was placed on the floor of the tank. This was followed by equal volumes of salt water with successive equal increases in density until the twenty layers were in place. The fact that the initially stepped profile could be produced precisely was checked by marking one intemediate layer with dye. It was noted that the interface between the coloured and uncoloured layers remained very sharp and horizontal, indicating that mixing by any turbulence generated in the course of filling the tank was strongly suppressed by the stable stratification. After filling, the ring was taken carefully out of the tank. The mixing caused by this action was also found to be negligible in the same way. The tank was then left for about two and a half hours to allow molecular diffusivity to smooth out the stepped density distribution. The coefficient of

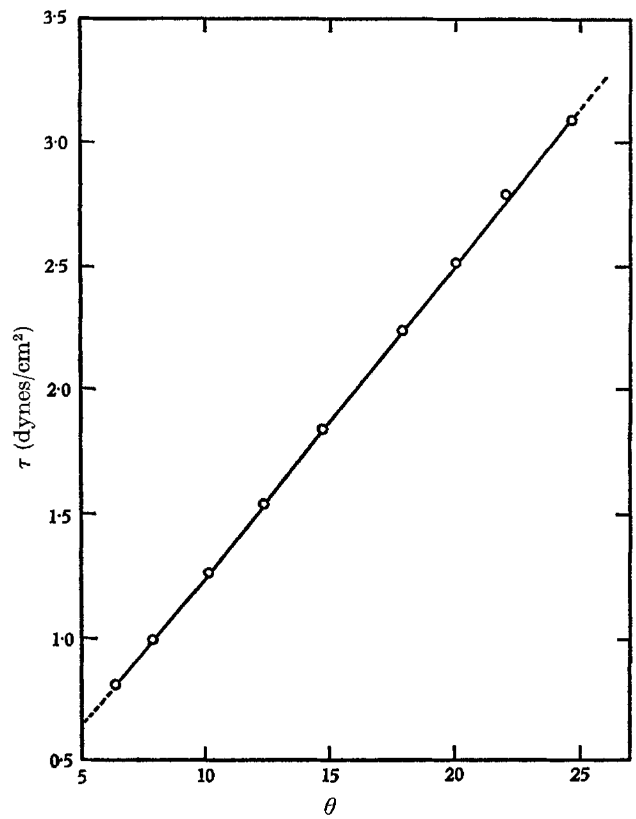

FIGURE 4. The calibration curve for the surface stress measurements. The stress τ_0 is found to be closely proportional to the angular displacement θ (arbitrary units) of the turntable.

diffusivity k_d of salt in water at the temperature of our experiment (13 °C in average) is nearly $1{\cdot}2 \times 10^{-5}\,\mathrm{cm^2\,sec^{-1}}$, and the corresponding length scale for diffusion $(k_d t)^{\frac{1}{2}} \sim 0{\cdot}33$ cm, which, according to Mowbray's (1967) results is sufficient to produce a density profile that is very nearly linear.

Three different initial density gradients were used in these experiments. In series C, $\partial\rho/\partial z = 1{\cdot}92 \times 10^{-3}$ g/cc/cm with a density range from 1·000 to 1·042; in series B, $\partial\rho/\partial z = 3{\cdot}84 \times 10^{-3}$ g/cc/cm, with a range 1·000 to 1·084 and in series D, $\partial\rho/\partial z = 7{\cdot}64 \times 10^{-3}$ with a range 1·000 to 1·092. In the last of these, only the upper half of the tank was useful, the lower part being filled with salt water of constant density ($\rho = 1{\cdot}092$).

The experiments were conducted in the following way. About 150 ml of a solution of Fluorescein dye was injected very gently into the water above the screen in a reasonably uniform manner around the annulus. Since the density gradient was already established (except very close to the surface where $\partial\rho/\partial z = 0$, because of the zero surface flux condition) the dye remained in this region. The motor and a clock were started simultaneously, the motor speed being increased smoothly until the desired torque was obtained. This took about 4 or 5 sec. After this, the speed was adjusted continuously by hand monitoring of a motor controller in order to maintain a constant torque T. The rate of increase of speed was largest initially and became smaller with time; after 1 min or so very little adjustment was needed. Each run was continued until the turbulent layer occupied about three-quarters of the water depth (about 16 cm, compared with the channel width of 22·8 cm) by which time the bottom effects and the influence of the finite width of the tank were certainly becoming appreciable. Since the the experiments were started with rapidly increasing torques, there was some uncertainty about the actual torque supplied to the water over the earliest few seconds of each run. Be that as it may, a nearly constant stress τ was applied to the top of the stratified water in this way by the screen and was maintained throughout the experiment.

The dye initially injected over the screen diffused downwards and spread out uniformly in the mixed layer, marking clearly its total extent. In order to confirm that the region marked by dye coincided with the turbulent mixed zone, a number of small particles (drops of a mixture of benzene and carbon tetrachloride) of varying densities was on several occasions dropped into the fluid before the run began. They distributed themselves throughout the fluid, each floating at its equilibrium level in the stratified fluid. By the time the coloured layer had developed to some depth, the particles originally in this region were concentrated either at the upper screen or at the interface between the coloured and uncoloured regions, lending support to our association of the edge of the coloured region with the density jump.

The circular drift in the upper turbulent layer resulted in a slight radial tilt of the interface, so that only the middle of the tank was illuminated by the sheet of light described earlier. Movie pictures were taken of the motion through the side window; these were read frame by frame to obtain the depth D of the turbulent layer as a sliding time average over periods from about 1·8 sec during the initial stages to 7 sec for the later stages. The motion of the wheel bearing special marks (figure 1, plate 1) was also visible in the same frames, so that the speed of the screen could be calculated. Observations on particles suspended in the upper layers gave little indication of any systematic secondary motion superimposed on the turbulence, though doubtless some existed. Corner eddies are well known in turbulent flow through a rectangular duct and there may have been some circulation induced by the slow drift around the annulus. The maximum angular velocity of the drift (0·05 rad/sec) was, however, small compared with other characteristic frequencies of the flow; one would anticipate that the circulation is correspondingly weak.

3. Some qualitative observations

It was originally hoped that measurements could be made of the entrainment in a homogeneous fluid, but this proved to be impossible. The system took 4 or 5 sec to generate the desired value of surface stress, by which time much of the region was already turbulent. The interface was so convoluted, and the entrainment so rapid, that it was not possible to estimate with any confidence the average depth of the interface as a function of time. For this reason, we reluctantly decided to confine our attention to fluids with initially linear stratification.

In these cases, the initiation of the motion was followed by the transient occurrence of a series of regularly spaced eddies very similar in character to those observed by Thorpe (1968). They sprang up, with their axes across the channel, within about 4 sec of starting, forming a beautifully regular pattern with a spacing of about twice the layer depth at that instant and persisting for 1 or 2 sec as they entrained uncoloured fluid. The larger the density gradient, the smaller appeared to be the amplitude and spacing. The pattern rapidly disintegrated and all regularity was lost. The fluid that had been entrained by the vortices was diffused throughout the upper layer and the interface re-established itself as an irregular and unsteady surface distorted evidently by the billows in the turbulence above. As time went by and the magnitude of the density jump across the interface increased, the appearance of the surface became less like that of the turbulent interface in a homogeneous fluid (but without the massive and large-scale convolutions), and more wavelike, with upward cusps where whisps of fluid were detached by the developing flow. This kind of surface appeared to be well established in our experiments by the time the depth of the layer reached 3 or 4 cm; it persisted as the motion went on, the amplitude of the 'waves' decreasing and their scale increasing as the layer became deeper and deeper. The rate of penetration continually decreased, as figure 5 shows, but the boundary between the turbulent region (marked by dye) and the non-turbulent region remained sharp and distinct. During this stage of the experiment, there was no discernible systematic motion below the interface, suspended drops of the mixture of benzene and carbon tetrachloride moving only slightly and irregularly in response to the motion of the interface itself.

Though a systematic study of the structure of the turbulent layer will be reported later, some preliminary observations were made of the distribution of mean velocity of this region. A vertical crimped wire was used to produce a line of bubbles by means of a voltage pulse. The movement of this line indicated that the mean velocity varied most rapidly near the screen and near the entrainment interface, being almost constant in the central region, where the velocity was typically about half that of the screen.

All of the experimental results considered here are concerned with this stage of the process, in which interfacial irregularities were evident and the depth of the mixed layer did not exceed about 60 per cent of the channel width or depth. On several occasions we continued to observe for a much greater time (20 or 30 min). By this time, the interface had penetrated to the lower quarter of the tank, and the rate of increase of depth of the layer was becoming very small indeed. The motion of the interface was restricted to very small amplitude, smooth undulations; but below the coloured layer, however, slow laminar drifting motions gradually became evident. The drift velocity was of the order 0·1 to 0·2 cm/sec, and the depth of the region involved gradually increased. Evidently, the density jump had become so large that entrainment virtually ceased, yet the fluid below the interface was set in motion by the purely viscous stress across it, the rate of diffusion of momentum by molecular viscosity being greater than that of salt by molecular diffusion.

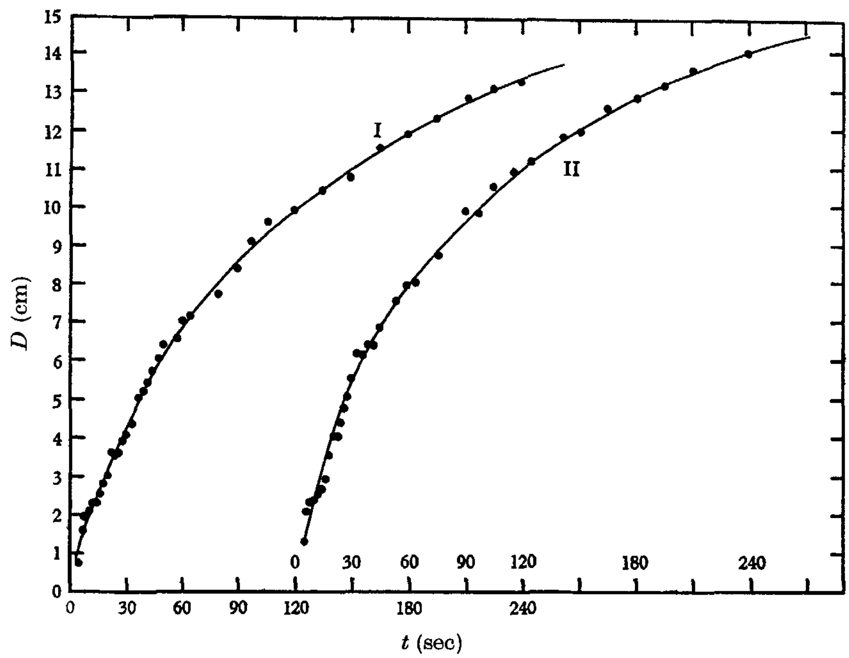

FIGURE 5. Typical variations in depth D of the mixed layer with time. Curve I: $d\rho/dz = 0{\cdot}00192$, $\tau_0 = 0{\cdot}995$ c.g.s. Curve II: $d\rho/dz = 0{\cdot}00384$, $\tau_0 = 2{\cdot}12$ c.g.s. Curve II is shifted to right by 120 seconds.

4. The rate of entrainment

The overall Reynolds number of the turbulent layer UD/ν increased in these experiments as the layer developed, as a result both of the increase in layer depth and of the speed U of the screen required to maintain constant stress. By the time the regular vortices had disintegrated, it was characteristically 5×10^3, rising to a value of order 2×10^4 by the time the screen velocity U was approaching its maximum. Though these values are not as large as one might desire, they are likely to be sufficient to ensure a turbulent flow whose large-scale structure is almost independent of the molecular viscosity. The properties of the turbulence should then be characterized simply by the friction velocity $u_* = [\tau/\rho]^{\frac{1}{2}}$ and the layer depth D.

Figure 5 is typical of the primary results obtained in this apparatus by analysis of the photographs. A total of eleven such runs was made successfully and in each of them, the rate of erosion decreased gradually as the layer depth and the density jump across the interface increased. The unsteadiness in the location of the interface is reflected in the scatter of the points shown in figure 5, each of which represents a sliding average as described earlier. It is not feasible to use a longer average in time, since the motion itself is unsteady, so that to obtain the *rate* of erosion, a continuous curve was faired through the experimental points and then differentiated numerically. Because of the uncertainties in τ at the beginning of each experiment and the possible bottom and side effects when D was large, only those measurements made in the interval between twenty seconds after starting and the time at which the depth was about 13 cm (55 per cent of the width) were used in the subsequent analysis.

If the entrainment process is independent of molecular viscosity and molecular diffusivity, the entrainment coefficient E, the ratio of the entrainment velocity u_e to the friction velocity u_* characterizing the flow, should be a function of the fractional change in buoyancy across the interface $g\,\delta\rho/\rho_0$, the friction velocity itself and the depth D of the layer, characterizing the scale of the turbulence. These last three variables can be grouped in a dimensionless form as an overall Richardson number of the layer of the kind defined by Ellison & Turner (1959)

$$Ri_0 = g\,\delta\rho\,D/\rho_0 u_*^2. \tag{3}$$

Since the upper layer was presumed to be well mixed (some evidence to that effect being offered by the observed uniformity of the dye throughout this region), the density jump across the interface,

$$\delta\rho = \frac{1}{2}\left(\frac{\partial\rho}{\partial z}\right)_0 D, \tag{4}$$

where $(\partial\rho/\partial z)_0$ is the initial density gradient. The overall Richardson number was therefore taken as

$$Ri_0 = \frac{g\,(\partial\rho/\partial z)_0\,D^2}{2\rho_0 u_*^2}. \tag{5}$$

On dimensional grounds, then,

$$E = \frac{u_e}{u_*} = f(Ri_0). \tag{6}$$

The data obtained were plotted in this way; the results are shown in figure 6. Although some scatter is evident, there seems to be little systematic departure from the form (6). In his experiments involving mechanical agitation in a tank, Turner (1968) found that, when the effects of molecular diffusion could be neglected, u_e was proportional to Ri^{-1}, where the velocity used to specify the Richardson number is defined in terms of the depth of the stirred layer and the agitation frequency. It is not possible to compare with any confidence the numerical values of the Richardson numbers in the two sets of experiments, because of the different parameters used in their specifications, but one would expect the functional forms to be the same provided the same processes were involved in each case. A slope (-1) is indicated in figure 6 and it is evident that it is perfectly consistent with the present results, though a 'best fit' slope might be incrementally less. If, however, we do adopt the relation found by Turner, these results enable us to specify completely the entrainment rate in terms of the friction velocity u_* acting at the surface:

$$E = \frac{u_e}{u_*} = 2\cdot5\,(Ri_0)^{-1} = 2\cdot5\,\frac{\rho_0 u_*^2}{g\,\delta\rho\,D}, \tag{7}$$

the numerical constant being uncertain to within about 30 per cent.

When the stratification is maintained by salt, this relation appears to hold over Richardson numbers between about 20 and 300. Over this range it appears that the direct influence of molecular diffusivity on the entrainment rate is small: Turner (1968) has shown clearly that when these do become important, the entrainment rate is reduced and decreases more rapidly with Ri, approximately as $Ri^{-\frac{3}{2}}$. There is no evidence for such a dependence in these experiments over the range of the values studied here.

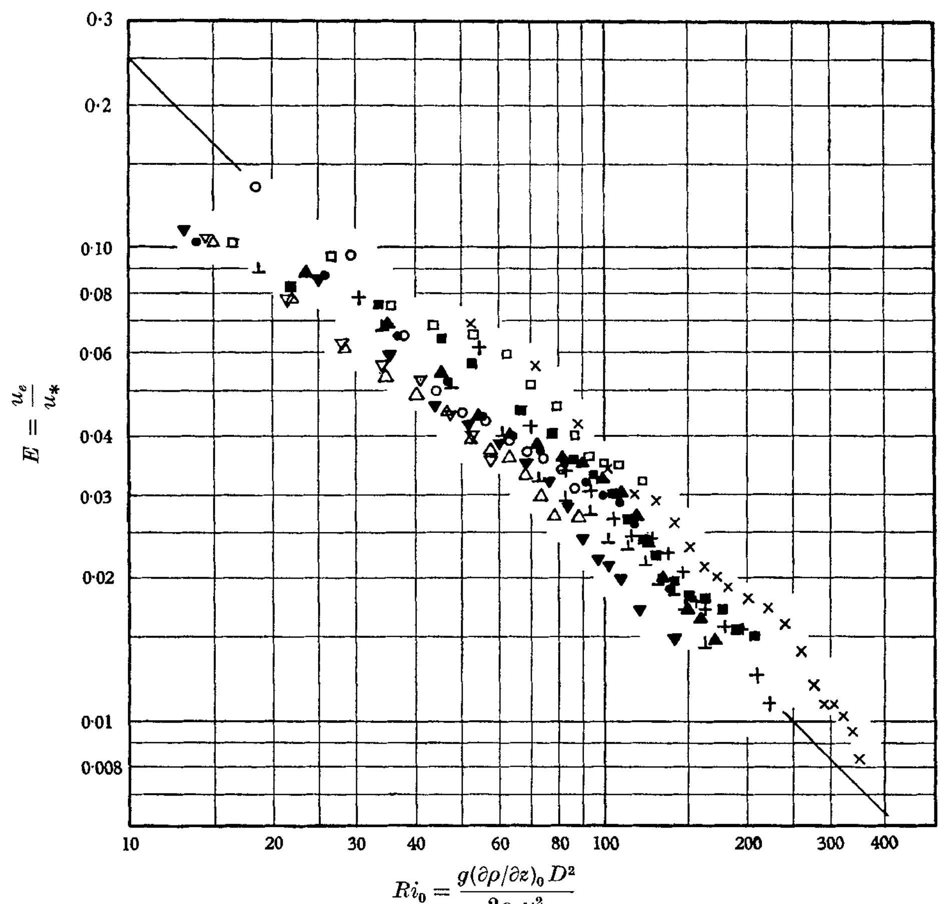

FIGURE 6. The entrainment coefficient E as a function of the overall Richardson number. The parameters represented by the symbols are indicated in the table below:

	$d\rho/dz = 1\cdot92\times10^{-3}$	$3\cdot84\times10^{-3}$	$7\cdot69\times10^{-3}$ c.g.s.
$\tau_0 =$ 0·995 c.g.s.	□	■	×
1·485	△	▲	
2·12	○	●	+
2·75	▽	▼	⊥

The result (7) can be used immediately to find the rate of increase with time of the surface mixed layer of the ocean under the influence of a wind. If the initial density distribution is $\rho(z)$, then when the mixing has proceeded to a depth D, the mean density in this region is

$$\bar{\rho} = \frac{1}{D}\int_{-D}^{0} \rho(z)\,dz,$$

and the density jump

$$\delta\rho = \rho(-D) - \bar{\rho}.$$

Thus

$$u_e = \frac{dD}{dt} = 2{\cdot}5\,\frac{\rho_0 u_*^3}{g\,\delta\rho D},$$

which can be solved for a given distribution $\rho(z)$ to give $D(t)$. For example, if

$$\rho(z) = \rho_0 - \beta z,$$

then it can be shown simply that

$$D(t) = u_* \left(\frac{15t}{N_0^2}\right)^{\frac{1}{3}},$$

where $N_0 = (g\beta)^{\frac{1}{2}}$ is the stability frequency.

5. The rate of increase of potential energy

The relation (7) is equivalent to a simple statement concerning the rate at which the potential energy of the density field is increased by the entrainment. In a time interval δt, a layer of thickness $\delta D = u_e \delta t$ and density $\rho + \delta\rho$ is entrained and replaced by fluid with density ρ; the mass difference is distributed throughout the upper mixed layer. Its centre of mass is therefore raised a distance $\frac{1}{2}D$, and the increase in potential energy per unit area is

$$\begin{aligned}\delta V &= \delta\rho\,\delta D g \tfrac{1}{2} D,\\ &= \tfrac{1}{2}\delta\rho g D u_e \delta t.\end{aligned}$$

Consequently, the rate of increase of potential energy

$$\begin{aligned}\frac{dV}{dt} &= \tfrac{1}{2}\delta\rho g D u_e,\\ &= 1{\cdot}25\rho u_*^3,\end{aligned} \tag{8}$$

where use is made of the result (7). The rate at which the potential energy increases is thus proportional simply to the cube of the friction velocity, and is independent both of the layer depth and of the density jump. The rate of turbulent energy dissipation per unit volume in a turbulent shear layer is proportional to u_*^3/D, so that the rate of increase of V is proportional to the rate of energy dissipation per unit area of the layer. The same kind of result was indicated by Turner (1968); while it provides the simplest statement of the energetics of the entrainment, it is at this stage far from obvious why it should be so. A fuller understanding must await the development of a comprehensive theory of the process.

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REFERENCES

COOPER, L. H. N. 1967 Stratification in the deep ocean. *Sci. Prog.* **55**, 73–90.

CROMWELL, T. 1960 Pycnoclines created by mixing in an aquarium tank. *J. Mar. Res.* **18**, 73–82.

ELLISON, T. H. & TURNER, J. S. 1959 Turbulent entrainment in stratified flows. *J. Fluid Mech.* **6**, 423–48.

MOWBRAY, D. E. 1967 The use of schlieren and shadow graph techniques in the study of flow patterns in density stratified fluids. *J. Fluid Mech.* **27**, 595–608.

ROUSE, H. & DODU, J. 1955 Diffusion turbulente à travers une discontinuité de densité. *La Houille Blanche,* 10[e] année, 522–32.

STOMMEL, H. & FEDEROV, K. N. 1967 Small scale structure in temperature and salinity near Timor and Minando. *Tellus,* **19**, 306–25.

THORPE, S. A. 1968 A method of producing a shear flow in a stratified fluid. *J. Fluid Mech.* **32**, 693–704.

TURNER, J. S. 1968 The influence of molecular diffusivity on turbulent entrainment across a density interface. *J. Fluid Mech.* **33**, 639–56.

TURNER, J. S. & KRAUS, E. B. 1967 A one-dimensional model of the seasonal thermocline. I. A laboratory experiment and its interpretation. *Tellus,* **19**, 88–97.

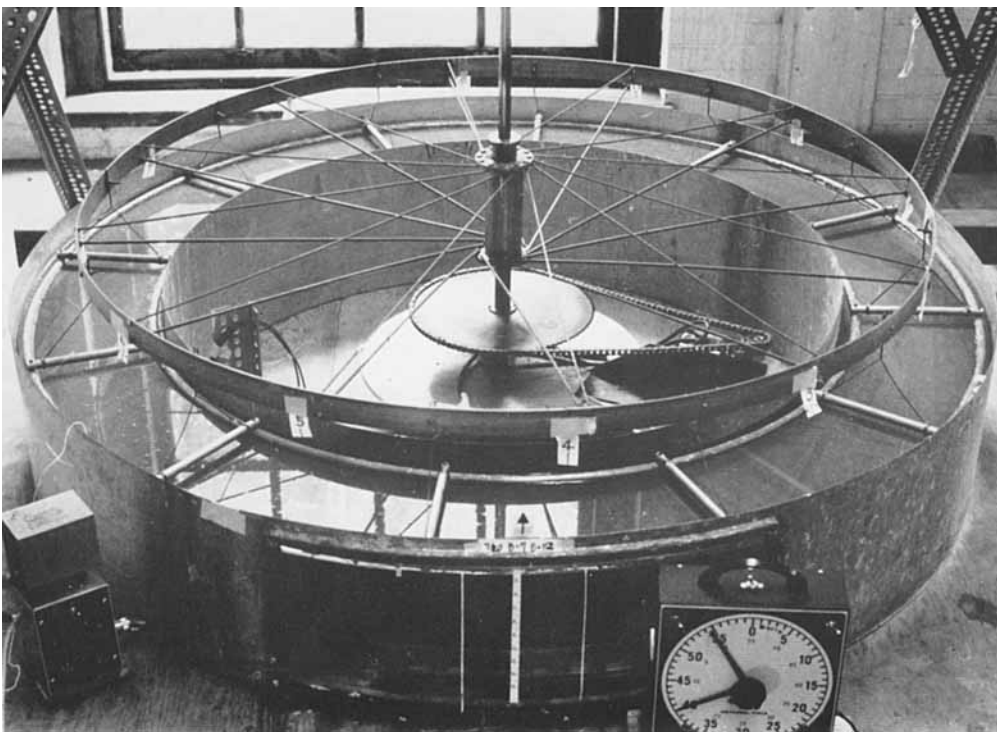

FIGURE 1. The experimental apparatus.